Lecture-1
Electro-Magnetic Field Theory

Introductory Concepts, Vector Fields and Coordinate Systems

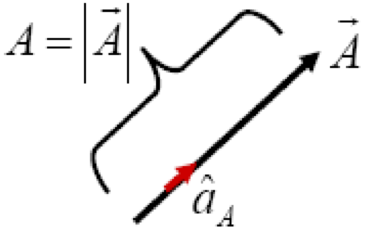
Scalar and Vector Fields

- **A scalar field is a function that gives us a single value of some variable for every point in space.**
  - **Examples: voltage, current, energy, temperature**
- **A vector is a quantity which has both a magnitude and a direction in space.**
  - **Examples: velocity, momentum, acceleration and force**

Vector Fields Explained

Vector has both <u>magnitude</u> and <u>direction</u> in space.

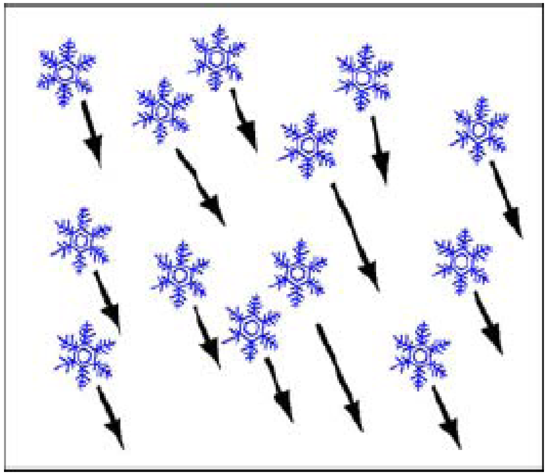
$$\vec{A} = \hat{a}_A A$$



$A = |\vec{A}|$ **Magnitude of the vector** $\vec{A}$

$\hat{a}_A = \dfrac{\vec{A}}{|\vec{A}|}$ **Unit vector in the direction of** $\vec{A}$

Falling Snowflakes



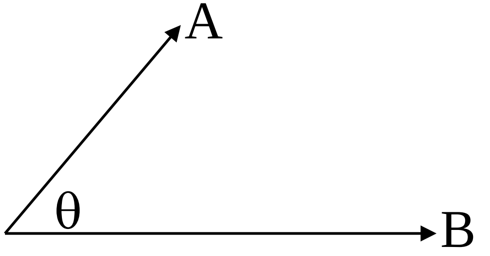
Velocity vector

Multiplication of vectors

- Two different interactions (what's the difference?)
  - Scalar or dot product :

$$A \cdot B = |A||B|\cos\theta = B \cdot A$$

    - the calculation giving the work done by a force during a displacement
    - work and hence energy are scalar quantities which arise from the multiplication of two vectors
    - if A·B = 0
      - The vector A is zero
      - The vector B is zero
      - $\theta = 90^\circ$



– Vector or cross product :

$$A \times B = |A||B| \sin\theta \, \mathbf{n}$$

- **n** is the unit vector along the normal to the plane containing A and B and its positive direction is determined as the right-hand screw rule

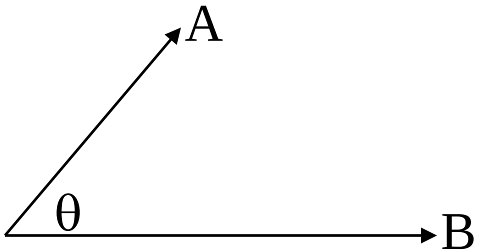
- the magnitude of the vector product of *A* and *B* is equal to the area of the parallelogram formed by *A* and *B*

$$A \times B = -B \times A$$

- if there is a force *F* acting at a point *P* with position vector *r* relative to an origin *O*, the moment of a force *F* about *O* is defined by :

- if A x B = 0
  - The vector A is zero
  - The vector B is zero
  - $\theta = 0^{\circ}$

$$L = r \times F$$



Commutative law :

$$A \cdot B = B \cdot A$$

$$A \times B = -B \times A$$

Distribution law :

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$A \times (B + C) = A \times B + A \times C$$

Associative law :

$$A \cdot BC \cdot D = (A \cdot B)(C \cdot D)$$

$$A \cdot BC = (A \cdot B)C$$

$$A \times B \cdot C = (A \times B) \cdot C$$

$$A \times (B \times C) \neq (A \times B) \times C$$

Unit vector relationships

- It is frequently useful to resolve vectors into components along the axial directions in terms of the unit vectors *i*, *j,* and *k*.

$$i \cdot j = j \cdot k = k \cdot i = 0$$
$$i \cdot i = j \cdot j = k \cdot k = 1$$

$$i \times i = j \times j = k \times k = 0$$
$$i \times j = k$$
$$j \times k = i$$
$$k \times i = j$$

$$A = A_x i + A_y j + A_z k$$
$$B = B_x i + B_y j + B_z k$$

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$
$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$